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Contents.

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THE EXPANSION OF THE UNIVERSE.

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ASTRONOMER ROYAL.

(Presidential Address, read at the Annual Meeting of the Society, on 26th July, 1933.)

No apology, I think, is needed for the subject which I have chosen for my Presidential Address. The theory that space — or the Universe — is expanding, has been put forward within the last few years to account for the observed motions of recession of the distant spiral nebulae or galaxies. At first sight, the explanation of these motions is elegant, simple and satisfying, but it is bound up with difficulties which have not yet been fully elucidated. The acceptance of the theory, at least in the form in which it is generally given, seems to necessitate the abandonment of the time scale of the evolution of the stars which was generally accepted before the development of the theory.

I propose first to discuss briefly the observational evidence upon which the theory has been built, and then to pass on to some consideration of the theory itself, the difficulties it involves and possible ways by which these difficulties may perhaps be avoided. The subject is a large one and it is not possible within the compass of a brief address to deal at all fully with all the problems which are suggested.

The pioneer work on the determination of the radial velocities of the extra-galactic nebulae was done by Dr. V. M. Slipher, Director of the Lowell Observatory, Flagstaff, Arizona, who was awarded this year the Gold Medal of the Royal Astronomical Society. Dr. Slipher was the first to make a determination of the radial velocity of a spiral nebula and altogether the velocities of some forty spiral nebulae have been determined at Flagstaff, exposures of very many hours having been required to obtain measurable spectra of most of these objects. The work has been extended to fainter objects at the Mount Wilson Observatory, using the 100-inch reflector in conjunction with very rapid spectrographs of small scale and of special design. One spectrograph used for this purpose has a specially designed objective with a focal ratio of 0·6 and a scale of 875 angstroms to the millimetre. The measurement of the spectral shifts of the lines, upon which the determination of the line-of-sight velocity is based in a spectrum of such small dispersion is necessarily somewhat uncertain, but the displacements which are observed in the spectra of the very faint spirals are so big that the somewhat large errors inherent in these measures amount to less than 2 per cent. of the total displacement. For a small spiral in Leo, with a velocity of about 20,000 kilometres per second, the displacement of the spectral lines is of the order of 300 angstrom units. The lines in the spectrum, upon whose displacement the determination of the velocities is usually based are the two lines, due to ionised atoms of calcium, at wave-length 3,968 and 3,933 angstroms — the so-called H and K lines of calcium. It is not without interest to note that these lines normally appear in the region of the spectrum where the intensity begins to fall rapidly off on account of absorption in the optical system. In the spectra of the rapidly moving spirals the lines are shifted so far from their normal positions in the direction of increasing wave-lengths that they fall in a region of the spectrum of greater intensity. The exposures necessary to photograph the lines are therefore appreciably shorter than would be necessary if the lines occurred in their normal positions. As a result of the observations which have been made at Mount Wilson, in conjunction with the earlier observations at Flagstaff, the velocities of about one hundred extra-galactic nebulae are now known. With few exceptions, the velocities are large and are in the outward direction.

It is found that, in general, the fainter the extra-galactic system the greater is its velocity of recession and a correlation between the integrated magnitudes of the systems and their distances is suggested. The question now arises as to how the distances can be determined. Direct methods are ruled out, as the probable errors of the best modern parallax determinations are many times greater than the parallaxes of the nearest spirals. Indirect methods must therefore be employed. The nearer spirals can be resolved, at least partially, in photographs with large telescopes into discrete stars, amongst which a number of variable stars of the Cepheid type have been detected. For the stars which show this type of variation, there has been found to be a very exact correlation between the period of the variation and the intrinsic luminosity of the star. This property may be expected to be characteristic of Cepheid variation, wherever it occurs in the Universe. The determination of the periods of variation of the Cepheids in the nearer spirals therefore provides a means for deriving the intrinsic brightness of these stars and hence of the distances of the spirals, subject only to the uncertainty which still exists as to the error in the zero-point of the period-luminosity curve of Cepheid variation. Such an error will cause all the determined distances to be in error by the same percentage, which may possibly amount to 25 per cent.

Using the distances of six spiral nebulae found in this way together with those of the two nearest extra-galactic systems, the Large and Small Magellanic Clouds, Dr. Hubble, at Mount Wilson, has found that the absolute magnitudes of the brightest stars in these eight systems have a very small dispersion about a mean value of $-6{\cdot}1$. It is a reasonable assumption to make that the same value would be obtained for other systems, if their distances could also be determined from observation of Cepheid variables. There are some thirty additional nebulae in which individual stars can be detected but in which Cepheid variables have so far not been observed. Distances for these nebulae can be inferred on the assumption that the mean absolute magnitude of the brightest stars in each nebula has the value of $-6{\cdot}1$. It may be objected that stars seen in close juxtaposition or even clusters of stars, similar to the globular clusters of our own galactic system, might not be resolved in the photographs of these systems and that the use of such a method would not be free in consequence, from the possibility of serious error. The photographs taken with the 100-inch telescope provide the best material that is available and Dr. Hubble has examined these very critically and carefully and is confident that the selection of the brightest stars, which he has made the basis of the determination of the distances of these nebulae, is not open to this objection.

By these two methods, based on the period-luminosity relation for Cepheid variables and on the mean absolute magnitude of the brightest stars, the distances of some forty spirals were determined by Hubble and it was found that there were strong correlations between the distances and apparent diameters of the nebulae and between the distances and apparent magnitudes. The existence of these correlations indicates that the spiral nebulae are of the same order of size and of total intrinsic luminosity. There is a dispersion about the mean values, but the dispersion is sufficiently small to permit the distances of the fainter and more distant spirals, which cannot be resolved into stars in photographs even with the 100-inch telescope, to be determined with fair approximation. Though it is true that the distance of any individual nebula, determined in this way, may be somewhat uncertain, the mean values should be sufficiently accurate for statistical purposes. Of the two criteria of distances, apparent diameter and total apparent magnitude, Hubble preferred to use the latter as less subject to error. The diameters are liable to vary with the length of exposure and the type of telescope used. The central portion of a spiral nebula is the brightest and the extent to which the fainter outer portions appear on the photograph will depend upon the length of the exposure; the failure to record the outer faintest portions of the spiral will obviously influence the determination of the diameter of the nebula to a greater extent than that of its total magnitude. The determination of the distances of the more distant nebulae therefore depends upon the measurement of the integrated magnitudes of these nebulae, and distances of sufficient accuracy for most statistical purposes can thus be found for every system whose radial velocity has been measured. It may be noted that the distances, determined by any or all of these three methods, are based ultimately upon the period-luminosity law of Cepheid variation and are subject to a common percentage error, depending upon the error in the zero point of the period-luminosity curve.

The velocities measured by these methods are found to be predominantly velocities of recession, increasing in direct proportion to the distances of the nebulae. A few of the nearer ones have small velocities of approach. These few exceptions were somewhat puzzling at first, as the phenomenon of a general recession is otherwise so strongly marked. But it must be remembered that the observed velocities are velocities relative to the Sun, which is itself a member of what is doubtless a spiral system. This system is known to be rotating about a centre in the direction of the constellation of Sagittarius and the linear velocity of the Sun about the centre of rotation is of the order of 300 kilometres per second. The velocities of the spirals should properly be referred to the centre of our galactic system. When this is done the larger apparent velocities of approach are found to be due to the motion of the Sun itself. The few small velocities of approach which remain, after the velocities have been referred to the centre of our galactic system, can be attributed to proper-motions of the systems in question and of our own system, for it appears that the extra-galactic nebulae have proper motions (apart from, and in addition to, the general motion of recession) of the order of 100 kilometres per second.

The available material has been summarised by Dr. Knox-Shaw. Reliable distances and velocities are available for 52 isolated nebulae and for 32 nebulae in eight clusters. The mean values are given in the following table, in which the distances are expressed in megaparsecs (1 megaparsec = one million parsecs = 3·26 million light-years). The first portion of the table gives mean values for isolated nebulae; the second portion gives values for clusters of nebulae, on the assumption that the dispersion in the distances of the nebulae in any one cluster is small compared with the distances themselves.

VELOCITIES AND DISTANCES OF EXTRA-GALACTIC NEBULAE.

GROUPS OF ISOLATED NEBULAE.

Distance (megaparsecs).	*No. of Nebulae.*	*Mean Distance (megaparsecs).*	*Mean Velocity (Km/sec.).*
0·03 to 0·48	8	0·26	138
0·60 „ 0·91	9	0·76	438
0·96 „ 1·15	8	1·06	761
1·32 „ 1·58	8	1·44	855
1·66 „ 2·09	7	1·78	1,233
2·19 „ 5·01	7	3·37	2,076
5·75 „ 10·00	5	7·50	4,122

CLUSTERS OF NEBULAE.

Constellation.	*No. of Nebulae Measured.*	*Distance (megaparsecs).*	*Mean Velocity (Km/sec.).*
Virgo	6	2·5	860
Pegasus	5	10·0	3,850
Pisces	4	9·6	4,760
Cancer	2	12·6	4,940
Perseus	4	15·1	5,380
Coma	8	19·0	7,340
Ursa Major ..	1	29·5	11,870
Leo	1	44·7	19,640

It will be seen that, in each portion of the table, the velocities are approximately proportional to the distances. From the figures given, Dr. Knox-Shaw derives, by least-square solutions, for the coefficient of proportionality (expressed in kilometres per second per megaparsec) the value 584 for the isolated nebulae and 419 for the clusters of nebulae. It does not seem probable that the two groups of nebulae can actually have different rates of recession. The distances of the faint distant nebulae have been inferred from their apparent magnitudes on the assumption that, the absolute magnitudes having small dispersion, a good approximation is obtained by supposing that the absolute magnitude of each distant nebula is equal to the mean absolute magnitude of the nearer nebulae. Possibly the apparent magnitudes of the faint nebulae have been underestimated, the inferred distances consequently being too great; or possibly the average member of the distant clusters of nebulae is fainter than the average isolated nebula. Further observations will be necessary to settle which of these alternatives is the correct one; it is, however, known that the magnitudes of the nebulae hitherto determined are subject to somewhat large errors and improved determinations are much to be desired.

The table above contains the observational material upon which the theory of the expanding Universe has been built. It appears that the distant nebulae are all receding from us with velocities which increase proportionally to their distances. At first sight it would appear to follow from this observational result that our galactic system occupies a privileged position at the centre of the Universe. But a little reflection will make it clear that every other distance in the Universe (*e.g.*, the distance between any two extra-galactic nebulae) is also increasing at a rate proportional to that distance. Every system is receding, not from any particular centre, but from every other system. The two-dimensional analogy of the surface of an india-rubber balloon has been used to make this clearer. Suppose that marks are made on the balloon to indicate different galactic systems and that the balloon is then inflated. As the inflation proceeds, the marks move radially outwards but each recedes also from every other one and the mutual distances increase at the same rate as the radius of the balloon. The whole system of the galaxies may therefore be said to be expanding. This is a purely observational fact independent of any theory. Upon it has been based the theory of the "expansion of space" or, otherwise expressed, the expansion of the Universe.

Before passing on to theoretical considerations, it is well to consider what the expansion of the galaxies at the observed rate implies. It can easily be deduced that all distances are doubled in a period of approximately 1,300 million years. The age of the Earth now generally accepted is about 3,000 million years. Since the Earth was born, all distances have more than quadrupled in size. Going backwards some millions of years more, there would have been a much denser concentration of the matter in the Universe than at present. From many lines of argument, we have been led to conclude that the stars have gone through a slow process of evolution, extending backwards for millions of millions of years. If, however, the Universe has been expanding at the present observed rate throughout this period, the distances of the extra-galactic nebulae would be enormously greater than they are observed to be. It is natural, therefore, that one should look round for some way of escape from this impasse.

We must come now to the consideration of the theory of expanding space. In 1917, Einstein announced his generalised theory of relativity by means of which gravitation was for the first time given a natural place in the scheme of things. In this theory there is formulated a set of "field-equations," which have to be satisfied throughout space. Assuming the Universe to be homogeneous and isotropic (so that any matter in the Universe is supposed to be uniformly distributed) two solutions of the field-equations were obtained, by Einstein and de Sitter respectively. At that time, only static, *i.e.*, non-expanding, solutions were thought of. One of these solutions (Einstein's) had a finite density of matter, but was truly static, for no systematic motions were possible in it and it did not expand. The other solution (de Sitter's) contained no matter at all, though motion was possible in it. Thus one contained matter but no motion, the other motion but no matter. Both these universes are closed or finite, so that if a person travelled on indefinitely in the same direction, he would ultimately return to the starting point. The space in both has positive curvature. It is of no use to attempt to picture the curvature of space; it is sufficient for our present purpose to state that mathematically there are three possible kinds of three-dimensional space, of which the curvatures are respectively positive, negative or zero. The latter is the ordinary flat or Euclidean space, which was the type of space visualised by Newton.

Though both these universes were static, in the sense that the Universe would remain unchanged for any length of time, in de Sitter's universe there was expansion in the sense that there would be an apparent recession of remote objects. Since, however, this universe is empty, there would be nothing to show expansion. It might be thought that the actual Universe is a good approximation to an empty universe, for if the matter which it contains were uniformly spread out there would only be an average of something like four hydrogen atoms to every cubic foot. The average density is thus only about the one-billionth part of the highest vacuum which we can produce in our laboratories — surely a sufficient approximation to emptiness. But if we consider the static universe of the Einstein type, with a radius of curvature of two thousand million light-years, we find that the average density is not greatly in excess of the mean observed density in the actual Universe. Our actual Universe must therefore be regarded as nearly full, rather than as practically empty of matter. We are therefore forced to the conclusion that neither Einstein's universe in which there is no motion, nor the empty universe of de Sitter can be regarded as a satisfactory first approximation to the conditions of the actual Universe. Other solutions of the field equations of generalised relativity must consequently be looked for. Such solutions will necessarily be non-static. The pioneer investigation of possible solutions of this nature was made by A. Friedmann in 1922, and later investigations were made, more or less independently, by the Abbé Lemaître, Tolman and Robertson. These solutions require expansion or contraction of space to take place; but whereas only two static solutions are possible, it has been found that a great variety of non-static solutions exist. Is it possible by some observational tests to narrow down these solutions and to say that our Universe cannot possibly belong to certain classes of them?

We must now say something about a mysterious constant called the *cosmical constant,* and generally denoted by the Greek letter lambda which plays an important part in the theory of the expansion of space. The field-equations of the generalised theory of relativity when written in their most general form contain a term in which λ enters as a multiplying factor. The initial formulation of Einstein's theory does not contain this term multiplied by λ in the field-equations. For explaining the motion of the perihelion of Mercury, the gravitational deflection of light rays in passing near matter and the displacement of the spectral lines of the Sun — which form the three crucial phenomena of the theory of relativity, the term in the equations which contains λ is not necessary. But it was found that without introducing λ a static solution with positive curvature was not possible. The introduction by Einstein of λ was therefore an *ad hoc* assumption, made to enable a static solution to be obtained. As we now know that a static solution cannot correspond to the actual Universe, the reason for the original introduction of λ has disappeared.

In the static solutions, the curvature of the space was proportional to λ; the radius of the curvature of space for such a solution is therefore inversely proportional to the square root of λ and can be regarded as a natural unit of length. On this basis, Sir Arthur Eddington has built up a theory of great elegance and beauty. It is not possible to give the details of this theory here. The essential underlying idea is that the wave-equation of a proton or an electron determines the size of the atom and hence must contain the cosmical natural unit of length. The equation as written by the physicist contains explicitly only such constants as the mass and charge of the electron, the velocity of light, Planck's constant, etc. Eddington considers that λ, or the radius of space, must implicitly be involved in the equations and must therefore be present in disguise. He concludes that the total number of particles (electrons or protons) present in the Universe, N, must also enter into the equation, since the atom is interacting with a universe which has N degrees of freedom. He combines the radius R, with N in the form $R/\sqrt{N}$, which has the dimensions of a length and identifies it with the expression e^2/mc^2 which enters into the wave-equation of the electron. Here e denotes the charge of an electron, m its mass and c the velocity of light. He therefore identifies these two quantities and writes:—

$$\frac{R}{\sqrt{N}} = \frac{e^2}{mc^2} \quad . \; . \; . \; . \quad (1)$$

It is perhaps necessary to explain that this is not a deduction from strictly logical reasoning, but that it is arrived at by a process of intuition. Even though the basic idea underlying this equation is correct, it is not clear why a numerical constant should not be introduced.

From the relativity theory of the expanding Universe, Eddington finds a second relation:—

$$\frac{N}{R} = \frac{\pi}{2\sqrt{3}} \cdot \frac{c^2}{Gm'} \quad . \; . \; . \; . \quad (2)$$

where G is the constant of gravitation and m′ is the mass of the hydrogen atom. From these two equations R and N can be separately determined; knowing R we can determine λ and the limiting speed of recession of the galaxies. λ is necessarily found to be positive and the limiting speed of recession of the galaxies is found to be between 500 and 1,000 km. per second per megaparsec, in satisfactory agreement with the observed value. The theory is certainly a beautiful piece of speculation but I cannot convince myself that the fundamental assumptions that the radius of curvature of space and the number of particles of matter in the Universe must enter into the wave-equation of an electron are correct. Also the neglect to insert an unknown constant in equation (1) above, by means of which the agreement with the observed rate of recession of the galaxies is obtained, seems to be purely arbitrary.

According to Edington's theory, the cosmical constant, λ has a determinate positive value. We have mentioned above that there are many different theoretical non-statical solutions possible of the field-equations of generalised relativity. Solutions exist of various types in which λ may be positive, negative, or zero. λ does not now determine the curvature, which may be positive, negative or zero for each possible value of λ. The possible theoretical solutions may be grouped into three classes, which de Sitter calls the oscillating universes and the expanding universes of the first and second kind. The oscillating universes have been investigated by Dr. Tolman, of the Californian Institute of Technology. The radius of such a universe increases to a certain maximum value, then decreases to a small (theoretically zero) value and increases again, with a finite period of oscillation. In the expanding universe of the first kind, the radius starts by being initially small (theoretically zero) and continually increases to become infinitely large after an infinite time. In the expanding universe of the second kind, the radius has a certain minimum value at zero time and becomes infinite after an infinite time. De Sitter gives the following table, summarising the different types of solutions which are theoretically possible for various values of the cosmical constant and of the curvature:—

λ	*Curvature.*		
	Negative.	*Zero.*	*Positive.*
Negative ..	Oscillating	Oscillating	Oscillating
Zero	Expanding I.	Expanding I.	Oscillating
Positive ..	Expanding I.	Expanding I.	Oscillating Expanding I. Expanding II.

Thus we see that, if λ should be negative, only oscillating universes are possible. Expanding universes of the first kind are the only ones possible when is zero or positive, if the curvature is either negative or zero, and so forth. We see further that it is possible for the cosmical constant to be zero, and then either expanding universes of the first kind or oscillating universes are possible. Alternatively, the curvature can be zero (Euclidean space) in universes of both these types.

Unless we make some assumption such as Eddington has made, the problem is indeterminate. For we have three unknowns, the value of λ, the sign of the curvature and the scale (relation between the radius and the time); observation gives us two data, viz., the rate of expansion and the average density. One assumption will make the problem theoretically determinate. At present, we have no means of determining which type of solution corresponds to the actual universe.

An ingenious theory of a different type to account for the observed velocities of recession of the extra-galactic nebulae has recently been put forward by Milne. This theory is purely kinematic. Milne considers a swarm of particles, moving in straight lines with uniform velocity and without collisions or other interactions. Suppose at the initial time the particles are contained within a certain sphere. Then, subsequently, the outward moving particles will gradually move into the empty space outside and the faster moving particles will gain on the slower. Particles, which at the initial time were moving inwards, will soon reach the opposite boundary of the swarm and will thereafter be moving outwards. Thus, after the lapse of sufficient time, all the fastest moving particles will have an outward motion. At a later time, it will be found that the outermost particles are those which have the largest velocities and that the velocities will gradually decrease inwards towards the centre. If the original velocity distribution was a continuous one and included zero and very small velocities, the original sphere will always be occupied by some of the particles.

It will at once be apparent that this behaviour of a cloud of particles bears a very close resemblance to the observed behaviour of the extra-galactic nebulae. The velocity is proportional to the distance from the centre, exactly as observed for the nebulae. It should be noted, however, that whereas in the relativity explanation the ratio of velocity to distance is constant both in time and space, in Milne's kinematical explanation the ratio is merely the reciprocal of the time, measured from the epoch of the initial configuration in the past up to the present moment. The observed correlation between velocity and distance gives 2,000 million years as the time that has elapsed since the swarm commenced to disperse. Whether the constant of proportionality does or does not change with time cannot, of course, be checked by observation.

Milne's theory gives a very simple picture, which is easy to grasp, but encounters the same difficulty as the form of the theory adopted by Eddington, viz., that the age of the Universe is very much less than had been supposed and is of the same order of magnitude as geological time. By the age, we refer to the time measured from the initial epoch, when the various members of the Universe were closely condensed together. Neither theory permits one to go backwards in time beyond this initial epoch, which, if you so desire, you may call "the beginning of the world." But, unless current astronomical theories are hopelessly in error, the evolution of the stars has been going on for a period of several hundred times as long as the age of the Universe, according to the expanding Universe theory. So we have the paradox that, as de Sitter has expressed it, the stars are much older than the Universe!

Let us recall briefly how an age for the Sun of something like five million million years has come to be generally accepted by astronomers. Stars vary in size, density, mass and surface temperature. From a study of these characteristics, it has been found that they can be arranged in a single continuous sequence, according to their mass and luminosity so that the mass being given, the luminosity is known to a fair degree of approximation, and vice-versa. The stars are continually pouring out into space large quantities of energy and since, according to the theory of relativity, mass and energy are synonymous terms, the stars must continually be losing mass. It is therefore difficult to resist the conclusion that, when the stars are arranged in a series according to their mass and luminosity — so that the high luminosity stars have high mass and the low luminosity stars have low mass — they have been arranged in an evolutionary sequence. This is necessarily merely inference, for it is not possible to observe evolution taking place in any stars, with the possible exception of the novae. If, then, the assumption is made that the continued output of radiation from the stars is made possible by the annihilation of matter, their ages can be computed with fair accuracy. The age thus deduced for the Sun is about five million million years.

An alternative means of production of energy, which gives an age for the Sun appreciably greater than the totally inadequate age of about fifty million years derived by Lord Kelvin, is provided by the transmutation of hydrogen into heavier elements. The energy released by such a process is rather less than one per cent. of the energy which could be obtained from the total annihilation of matter; this would perhaps provide for sufficient energy for the age computed from Eddington's expansion of space theory. But we should then be compelled to abandon our ideas as to the evolution of the stars. The mass-luminosity relationship would then become nothing more than an observed relationship, with no evolutionary significance.

Whatever view we take about the time-scale, there are difficulties to be faced. Eddington has shown, from the consideration of the dynamics of our galactic system, that its form and construction, as we now observe it, is such that there are grave difficulties in assuming that it can have existed for the length of time required by the longer scale. On the other hand, it is difficult to account for the evolution of double stars and of star clusters, if we are restricted to the length of time required by the shorter scale. So no definite pronouncement can be made in favour of either the one scale or the other, though I think that most astronomers, myself included, would favour the longer time.

Is the longer time-scale, however, necessarily incompatible with the theory of the expanding Universe? It does not seem to me to be impossible to reconcile the one with the other. We have seen that there are three possible types of non-static universes, which have been designated the expanding universe of the first kind, the expanding universe of the second kind and the oscillating universe. The oscillating universe provides a way out of the difficulties with which we are faced. Reference to the table above will show that an oscillating universe is possible with either negative, zero or positive values for the cosmical constant, λ. If we take the view that λ ought never to have been introduced into the theory, having been brought in solely to avoid a difficulty which no longer exists, we can assume λ to be zero. The oscillating universe is then possible in space of positive curvature but is not possible in Euclidean space. If, on the other hand, we incline to the view that space is Euclidean, so that there is zero curvature, then the oscillating universe is possible provided that λ is negative. As a third alternative, we can follow Eddington and accept positive values for both λ and the space-curvature and we then find that an oscillating universe is again possible.

With the oscillating universe, there is no difficulty in accepting de Sitter's paradox that the stars are much older than the Universe. The evolution of the stars starts from the time when they were first condensed out of nebulosity and may have been in progress for many oscillations. The time scale as fixed by the present observed rate of recession of the spiral nebulae commences with the beginning of the present oscillation. We have seen that, in the oscillating universe, the radius at minimum has a small value, which is theoretically zero. The theory is based, however, on the assumption of a uniform distribution of matter which is merely an approximation to the observed universe. Actually the radius at minimum would be small but not zero. Expansion then commences and proceeds until the radius has reached a certain maximum value, after which contraction sets in. These expansions and contractions will theoretically go on indefinitely. The deduced period of oscillation is of the order of a few thousand million years, which is about the age of the solar system. You are all familiar with the theory that the planetary system around our Sun was born as the result of a close passage of another star to the Sun. No other theory of the origin of our planetary system, yet propounded, is free from serious objection. The one criticism that this theory has had to sustain is that the average distance apart of the stars is so great that the probability of an encounter of two stars, sufficiently close to give rise to large tidal distortions, is extremely small. There is a not unnatural hesitation in accepting unreservedly a theory of the origin of the solar system which ascribes to the system a condition which is more or less unique. But if at the time the system was formed the individual galaxies were so close that they penetrated one another, then the chance of an approach of two stars to one another, sufficiently near for birth to be given to a planetary system, is much increased and the solar system no longer need be regarded as something unique in our galactic system.

Sir Arthur Eddington has said that he is an evolutionist and not a multiplicationist and that for him there is no interest in doing the same thing over and over again. Others may take a different view. Oscillation is a very common phenomenon in Nature and one must to a large extent be guided by aesthetic considerations in accepting or rejecting any single one of the various solutions of the relativistic field-equations, which are mathematically possible.

The oscillating universe has possibly another claim to consideration. If the energy which the stars emit is derived from the annihilation of matter, we have apparently a one-way non-reversible process in operation; matter is being converted into radiation, energy is being degraded and entropy increased. The Universe can be compared to a clock which is running down, sooner or later to come to a complete stop. Many astronomers, I think, have hoped that there might be some way of escape from this conclusion. Such a way of escape seems to be provided by the relativistic thermodynamics, developed by Dr. Tolman. He has succeeded in enunciating the laws of thermodynamics in a relativistic form invariant for all transformations of axes. These generalised expressions of the laws of thermodynamics should therefore be applicable throughout space. Tolman finds that, with the law so expressed, reversible transformation of matter into radiation can take place without change of entropy of the Universe as a whole. If, on balance, matter is being converted into radiation then the radius of the Universe must be expanding. If an observer measures the entropy in the portion of the Universe in his immediate neighbourhood and takes no account of the expansion of space, he will conclude that there is a continual increase in the entropy in his neighbourhood. This is what is found by physicists now. Tolman further finds that a non-static oscillating model of the Universe is possible according to which the irreversible annihilation of matter, giving rise to radiation, would take place in the later stages of the expansion and the irreversible formation again of matter out of radiation would take place in the later stages of contraction. If this is so, it is just a matter of chance that we happen to be living at a time when the Universe is expanding and when there is a large net conversion of matter into radiation. It has been somewhat of a puzzle as to what happens to all the radiation which the stars have for millions of years been pouring out into space. On Tolman's theory this is no longer a difficulty. After the expansion has ceased and contraction has set in, the converse process will begin to take place. There will then be a large net conversion of radiation back into matter. If we had lived a few thousand million years hence, we might have been puzling over an explanation for the remarkable fact that all the distant nebulae appeared to be moving towards us and that the further away they happened to be, the faster they were moving.

The theory of the oscillating universe commends itself to me partly on aesthetic grounds and partly because it seems on the whole to provide the easiest way out of some of the difficulties with which astronomy is now faced. There are no doubt objections to it, as there are to any of the alternative theories. The one thing that is certain is that we have at present no definite criterion for asserting that any single one of the possible types of universe, which is permitted by the generalised theory of relativity, corresponds most closely to the Universe as it actually exists at the present time. With this in mind, I suggest that the theory that the Universe, though expanding at the moment, is really in a state of oscillation, does provide a way of escape from some of the more serious difficulties to which reference has been made.

PRIME NUMBERS.

By Lawrence Crawford, M.A. (Camb.), D.Sc. (Glasgow), F.R.S.E., F.R.S.S.Af.

Paper read to the Mathematical Section.

1. When I was asked to give a paper to this newly formed Mathematical Section I felt I could not refuse, as I am one of the veteran teachers of mathematics in South Africa. But I found it difficult to choose a subject. I wanted to give the members of the section some mathematics; at the same time I knew that some have not gone very far in mathematics and some have forgotten a good deal of what they had known, so that a paper entirely on advanced mathematics would be uninteresting. I told the Secretary, therefore, that I would speak on Numbers, and I have decided to speak on Prime Numbers. Everyone knows what prime numbers are: 2, 3, 5, 7, etc., numbers with no factors. I hope to give some results which are unfamiliar to many members and I shall devote some time to the solution of one of the great problems connected with these numbers. It has introduced into mathematics the Zeta function of Riemann. This function and the allied series of Dirichlet are the backbone of work on the theory of numbers and have been, and still are, the subjects of innumerable papers.

2. The prime numbers less than 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97—in all, 25. They proceed irregularly; there are four between 11 and 20, two in 31 to 40, 3 in 71 to 80, and only one in 91 to 100.

Except for 2 and 3, every prime is of the form $6m\pm1$. But any number of this form is not necessarily a prime; $6.4+1=25$, $6.6-1=35$, not primes.

There is no short rule by which it can be detected if a number is prime or not. To see if x is prime, every number less than $\sqrt{x}$ has to be tested as a factor. Note that if there is a factor bigger than $\sqrt{x}$, the other factor must be smaller than $\sqrt{x}$ as the product is x, so only numbers less than $\sqrt{x}$ need be tested as factors.

3. The number of primes is infinite; this result was proved by Euclid, Book 9, Prop. 20.

If p is a prime number, take $N=2.3.5.7\ \ldots\ p+1$, the product of all primes up to p with 1 added. N divided by any one of these primes leaves a remainder 1 and has therefore no one of them as a factor. If N is a composite number, then N must have a factor greater than p and 2, 3, 5, 7, etc., are not factors of it or they would be factors of N, so that this number must be a prime and is greater than p. If N is a prime number, it is a prime greater than p, so that in any case there exists a prime number greater than p. Hence whatever prime number p is known, we can always get a larger one.

This does not help us to find the larger one, for N need not be prime.

$2.3.5.7.11+1=2311$ and is prime.

$2.3.5.7.11.13+1=30031$ and on testing primes from 17 to 173 (the highest prime number less than $\sqrt{30031}$) it is found this number has a factor 59 and is not prime.

4. 2^n-1 can only be a prime if n is prime, but the converse does not follow. $2^7-1=127$, a prime; $2^{11}-1=2047=23\times89$, a composite number.

2^n+1 can only be a prime if n is of the form 2^k, but again the converse does not follow. Fermat in 1654 considered that the result was always true, but Euler disproved it about 1732. He showed that 2^n+1, where $n=2^5=32$, is 4294967297 which is 6700417×641. He also showed that 2^n+1, where $n=2^6$ or 2^{12} or 2^{36}, are not primes, the last one having as a factor the prime 2,748,779,069,441, the number itself having more than twenty thousand million figures. To the astronomers who deal in light years for distances between stars such large numbers are of course familiar.

5. A sequel to these expressions for prime numbers is the perfect number, a number equal to the sum of its factors; the two lowest are $6=1+2+3$, $28=1+2+4+7+14$. [*Note*, 1 is taken here as a factor, though in other work it is ignored.]

Euclid proved that if p is a prime and 2^p-1 is prime, then $2^{p-1}(2^p-1)$ is a perfect number and it has been proved that every even perfect number is of this form.

Hence every even perfect number ends with a 6 or an 8, for 2^p and 2^{p-1} are both even, taking $p>2$, hence the table:

Last figure in 2^p.	Last in 2^{p-1}	Last in 2^p-1	Last in product.
2	6	1	6
4	2	3	6
6	8	5	here 2^p-1 is not prime
8	4	7	8

The first four even perfect numbers are given by Euclid's formula for $p=2, 3, 5, 7$. The fifth was given about 1461 and is 33,550,336, others given then were incorrect. Every value of p does not give a perfect number by Euclid's formula, for note that 2^p-1 must be prime: the first nine even perfect numbers are given by $p=2, 3, 5, 7, 13, 17, 19, 31, 61$, the last one containing 37 figures.

Are there an infinite number of values of the prime p making 2^p-1 a prime? We do not know. Are there an infinite number of odd perfect numbers, or is there even one? We do not know. But Landau, who wrote three volumes on the Theory of Numbers after two volumes on Prime Numbers begs his readers not to linger over these two questions but to pass on to much more hopeful and fruitful problems.

6. I pass by Fermat's Theorem, $a^{p-1}-1$ is a multiple of p if p is a prime and a prime to p, and Wilson's Theorem, $(p-1)!+1$ is a multiple of p if p is a prime, and proceed to the great problem in prime numbers, to find $\pi(x)$ where $\pi(x)$ is the number of primes $<x$, or equal to x, where x is any given number.

In 1808 Legendre stated that for large values of x, $\pi(x)$ is approximately equal to $\dfrac{x}{\log x-B}$, where B is a certain numerical constant and $\log x$ is the natural logarithm, to the base e [2·71828 . . . .]. His original suggestion that $B=1{\cdot}08366$, based on tables extending only as far as $x=400{,}000$ was soon discarded, and later work dealt with $\dfrac{x}{\log x}$ or $\dfrac{x}{\log x-1}$.

A similar formula was proposed independently not long after by Gauss; he suggested $\dfrac{1}{\log x}$ as an approximation to the average density of distribution or the number of primes per unit interval, again when x is large, and so got as an approximation to $\pi(x)$ the function $\displaystyle\int_2^x \frac{du}{\log u}$. In later work this was replaced by $li x$, defined as the limit for $\eta=+0$ of $\displaystyle\left(\int_0^{1-\eta}+\int_{1-\eta}^{x}\right)\frac{du}{\log u}$, [Note that $\log u=0$ when $u=1$, hence the 1 is cut out of the range of integration and a principal value given], which differs from $\displaystyle\int_2^x \frac{du}{\log u}$ by approximately 1·04.

Neither Legendre or Gauss made clear the precise degree of approximation, but it is taken that they intended to imply the asymptotic equivalence of $\pi(x)$ to one of these functions, *i.e.* calling either of these functions $f(x)$, $\dfrac{\pi(x)}{f(x)}$ can be made to differ from 1 by as small a quantity as we please by taking x sufficiently great. This is denoted by $\pi(x)\sim f(x)$.

The central problem then was to prove $\pi(x) \sim \frac{x}{\log x}$.

Great advances in the solution of the problem were made about 1850 by Chebyshev (or Tschebyscheff). He introduced a new function $\psi(x)$ where $\psi(x)$

$=\log 2+\log 3+ \ldots\ldots +\log q_1$, where q_1 is highest prime below or equal to x

$+\log 2+\log 3 \ldots\ldots +\log q_2$, where q_2 is highest prime such that $q_2{}^2<x$ or $=x$

$+\log 2+\log 3+ \ldots\ldots +\log q_3$, where q_3 is highest prime such that $q_3{}^3<x$ or $=x$

$+ \ldots\ldots\ldots\ldots\ldots$

In each line the number of terms diminishes and the series stops, for as soon as $2^m>x$, there are no terms in the line corresponding to the number m and the series ends with the previous line. Note $\psi(x)$ is the logarithm of the L.C.M. of the numbers 1, 2, 3, ... x.

He proved that the theorems $\pi(x) \sim \frac{x}{\log x}$, $\psi(x) \sim x$ hold together, if one is true, so is the other. Other writers worked, like him, with $\psi(x)$ and the latest conclusion was that $1{\cdot}04423+\eta> \frac{\psi(x)}{x}> {\cdot}95695+\eta'$, when x is sufficiently large and η,η' are very small when x is sufficiently large.

This does not, however, prove that for x very large, the limit of $\frac{\psi(x)}{x}$ is 1, for the result is consistent with the hypothesis that $\frac{\psi(x)}{x}$ oscillates between two finite limits without tending to a single value when $x=\infty$.

It was in 1896 that Hadamard and Vallée Poussin proved conclusively that $\pi(x)$ is represented more accurately by li$x$ than by $\frac{x}{\log x-B}$ whatever value was given to B, that the most favourable value of B is 1, and that the limit of $\frac{\pi(x)}{\text{li}x}$ is 1 when $x=\infty$ Their proofs depended on the ζ-function of Riemann and no proof that $\psi(x) \sim x$, or $\pi(x) \sim \text{li}x$, or $\pi(x) \sim \frac{x}{\log x}$ has been given without the use of that function.

7. To show how nearly $\pi(x)$ and li$x$ approach one another, a table is given giving their values and their ratio :

x	$\pi(x)$	li$x$	$\frac{\pi(x)}{\text{li}x}$
1,000	168	178	0·94
10,000	1,229	1,246	0·98
100,000	9,592	9,630	0·996
1,000,000	78,498	78,628	0·9983
10,000,000	664,579	664,918	0·9994
100,000,000	5,761,455	5,762,209	0·99986
1,000,000,000	50,847,478	50,849,235	0·99996

It will be noticed in the table that $\pi(x) \angle \text{li}x$, but in 1914 Littlewood proved that if we go far enough we come to numbers where $\pi(x) > \text{li}x$ but such numbers are far beyond what tables we have of $\pi(x)$. His result is :

$\pi(x) - \text{li}x > \frac{x^{\frac{1}{2}}}{\log x}\{-1+\tfrac{1}{2}(1-\epsilon)\log\log\log x - \epsilon\}$ where ϵ is some number between 0 and 1. Provided x is large enough, log log log x can be made large enough to make the right hand side positive and therefore $\pi(x) > \text{li}x$. But we cannot say this is true for $x = 10^{700}$, a number of 701 figures, for then log $x = 1612$, log log $x = 7 \cdot 3$, log log log x is approximately 2. [Note, logarithms are to base e.]

$$\therefore\ \pi(x) - \text{li}x > \frac{x^{\frac{1}{2}}}{\log x}\{-1+\tfrac{1}{2}(1-\epsilon)\,.\,2-\epsilon\}$$

$$\therefore\ \pi(x) - \text{li}x > \frac{x^{\frac{1}{2}}}{\log x}(-2\epsilon)$$

and we cannot say $\pi(x) - \text{li}x$ is positive.

The results for $\frac{x}{\log x}$ and $\frac{x}{\log x - 1}$ may be given for comparison :

x	$\frac{x}{\log x}$	$\frac{x}{\log x - 1}$
1,000	109	122
10,000,000	620,420	661,460
1,000,000,000	48,255,000	50,702,000

8. Riemann's ζ-function is defined as follows :

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \ldots\ldots + \frac{1}{n^s} \ldots = \sum_1^\infty \frac{1}{n^s}$$

and the allied series are of the type $\sum_1^\infty \frac{a_n}{n^s}$

$\zeta(s)$ is finite for $s>1$; for $s=1$ or $\angle 1$, the value of the right-hand side can be made larger than any quantity we please on taking a sufficient number of terms.

$$\zeta(s)\times\frac{1}{2^s}=\frac{1}{2^s}+\frac{1}{4^s}+\frac{1}{6^s}+\ldots.$$

$\therefore\ \zeta(s)\left(1-\frac{1}{2^s}\right)=\frac{1}{1^s}+\frac{1}{3^s}+\frac{1}{5^s}+\ldots.$, all terms like $\frac{1}{(2m)^s}$ going out

$\therefore\ \zeta(s)\left(1-\frac{1}{2^s}\right)\frac{1}{3^s}=\frac{1}{3^s}+\frac{1}{9^s}+\frac{1}{15^s}+\ldots.$

$\therefore\ \zeta(s)\left(1-\frac{1}{2^s}\right)\left(1-\frac{1}{3^s}\right)=\frac{1}{1^s}+\frac{1}{5^s}+\ldots.$, all terms like $\frac{1}{(3m)s}$ going out

Proceed in this way and we find

$$\zeta(s)\left(1-\frac{1}{2^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{5^s}\right)\ldots.=1$$

i.e., $\zeta(s)=\prod_{p=2}^{\infty}\frac{1}{1-\frac{1}{p^s}}$ where the product extends over all prime numbers p.

Hence $\log\ \zeta(s)=-\Sigma\log\left(1-\frac{1}{p^s}\right)$

$$=\Sigma\left(\frac{1}{p^s}+\frac{1}{2}\cdot\frac{1}{p^{2s}}+\frac{1}{3}\cdot\frac{1}{p^{3s}}+\ldots.\right)$$

$\therefore$ differentiating, $\frac{\zeta'(s)}{\zeta(s)}=-\Sigma\{\frac{\log p}{p^s}+\frac{\log p}{p^{2s}}+\frac{\log p}{p^{3s}}+\ldots.\}$, the summation being over all primes.

This may be written, $\frac{\zeta'(s)}{\zeta(s)}=-\sum_{1}^{\infty}\frac{f(n)}{n^s}$ where the summation is over *all* numbers, but $f(n)=\log p$, if n is a power of a prime number p, and 0, if n is not a power of a prime.

Here $\Sigma f(n)$ for numbers up to x is the former $\psi(x)$ and so a connection is made between $\zeta(s)$ and $\psi(x)$.

The full proof that $\psi(x)\sim x$ depends on a study of $\zeta(s)$ as a function of a complex variable, $s=\delta+it$, where δ and t are both real. The necessary theorem is that $\zeta(s)$ cannot be zero for $s=1+it$, whatever be t. This is proved from the above result for $\log\ \zeta(s)$ and the formula, $3+4\cos\theta+\cos 2\theta>0$, or $=0$ which holds for all angles θ, since this expression is $2(1+\cos\theta)^2$.

9. To prove that $\psi(x)\sim x$, we begin with $\psi_1(x)=\int_0^x\psi(u)du$ and prove $\psi_1(x)$ connected with $\zeta(s)$ by a contour integral. Then the theorem is proved that $\psi_1(x)\sim\frac{1}{2}x^2$, and in doing this contour integration is again used and the theorem that $\zeta(1+it)\neq 0$ is used in integrating along parts of the line $\delta=1$.

From $\psi_1(x) \sim \frac{1}{2}x^2$, the result $\psi(x) \sim x$ is derived and so $\pi x \sim \frac{x}{\log x}$ is proved.

It is a remarkable result that the complete proof of $\pi(x) \sim \frac{x}{\log x}$, or $\pi(x) \sim lix$, should depend on the theorem that $\zeta(s)$ has no zero of the form $1+it$, an instance of how work on the function of a complex variable comes in to so much of advanced work in mathematics.

In proving $\psi_1(x) \sim \frac{1}{2}x^2$, another result is obtained, $p_n \sim n \log n$ where p_n is the nth prime. p_n and $n \log n$ are far from equal for ordinary values of n; if n is 25, p_n is 97 and $n \log n$ is about 80.

More detailed work on the zeros of $\zeta(s)$ leads to results which give approximate values for $\psi(x)-x$, $\pi(x)-lix$, for x large, such as the result in §7. The work is, however, very complicated. Even yet the work on $\zeta(s)$ is not complete for Riemann's hypothesis that a system of zeros of $\zeta(s)$ lies on the line $\delta=\frac{1}{2}$, *i.e.* are of the form $\frac{1}{2}+it$, has not yet been proved or disproved.

10. Other results connected with prime numbers are: every large odd number is the sum of three primes, almost all even numbers are the sums of two primes, but these and similar theorems all depend on work arising out of the ζ-function. $x^m+y^m=z^m$ is impossible for m prime, if x, y, z are all numbers different from zero and $m>2$, is Fermat's last theorem. It has been proved for $m<257$ and for certain classes of prime numbers, but the complete proof has not yet been given.

11. In conclusion, I wish to express my special debt in making up this paper to the *Encyclopaedia of Mathematics* (the French edition was the one used) and to Ingham's book on the Distribution of Prime Numbers, *Cambridge Mathematical Tract*, No. 30.

Note.—A paper has just been published in the Journal of the London Mathematical Society by Mr. S. Skewes, a member of this Astronomical Society, giving an approximate value of x for which $\pi(x) > \text{li}x$.

REVIEWS.

"The Makers of Astronomy." By Hector MacPherson. [Oxford: At the Clarendon Press, 1933. 7s. 6d.]

All students of science should study the history of science. This is specially true in the case of astronomy—for not only is the history of individual astronomers interesting in itself, but modern results are easier to understand and the possible lines of future advances indicated by a study of the lines of development in the past. Rev. Hector MacPherson is well known as a writer on astronomical history and particularly that of the Herschels. This new book describes the advances made in astronomy, with short histories of the most important astronomers from Copernicus onwards. At the present time, when we in South Africa are preparing to celebrate the centenary of the arrival of Sir John Herschel, it is useful to have a book giving a whole chapter to the Herschels, and indeed the book shows how the modern conception of the structure and extent of the universe is the direct outcome of the observations and ideas of Sir William Herschel. There are many astronomers whose lives are not recorded in this book, but it is interesting to have short biographies of several, specially German astronomers, whose history is not too well known. This volume is different from most biographical books in being up to date and dealing with many workers of the present day. It is beautifully printed and may be recommended to anyone who would like to spend a few pleasant hours reading about astronomers and their work.

"The Composition of the Stars." By H. N. Russell. [Oxford: At the Clarendon Press, 1933. 2s.]

Prof. H. N. Russell, in the Halley Lecture at Oxford this year, described the information as to the composition of the stars (or at least their atmospheres) which could be deduced from a detailed examination of their spectra. The problem is a complicated one, for the strength of the lines is by no means proportional to the amount of the elements present. Sometimes the principal spectral lines have a wavelength less than 2,900 A and so are cut off by our atmosphere. Sometimes they are far in the infra red and difficult to observe. Or the temperature may be so high or the pressure so low that the element may be completely ionized and the spectrum of the neutral element absent. Again, the presence of electrons due to ionisation of other elements may make the atmosphere too opaque for certain lines to be strong enough. Considering the difficulties, it is remarkable how much can be deduced from stellar spectra as to the elements present, the temperature and pressure of the surfaces. We must not forget, too, that displacement of lines due to motion give us radial velocities and may give axial rotation (as it does for a few eclipsing binaries). It may be interesting to quote a few results for the sun: "Definite proof has been found of the presence of 58 elements, and more or less certain evidence of four more. Eighteen show no signs of their presence, while for the remaining ten the spectra are not well enough known for a decision." Of the eighteen elements whose lines are missing from the solar spectrum, no less than fourteen have their principal lines in the region for which our atmosphere is opaque. Of the other four—caesium, bismuth, rhenium and radium—the last two are rare on the earth; caesium has its ultimate lines in the infra red, it is the easiest to ionise of all the elements, and must be completely ionised even in sunspots. Bismuth must be actually very rare on the sun. Seven elements make up 96% by weight of the solar metals, and these are exactly the seven principal constituents of our terrestrial rocks.

This book is warmly recommended to those interested in making the maximum deductions from the information contained in stellar spectra.

"The Place of Observation in Astronomy." By H. H. Plaskett. [Oxford: At the Clarendon Press, 1933. 2s.]

This pamphlet constitutes the inaugural address of Dr. H. H. Plaskett as Savilian Professor of Astronomy at Oxford. It surveys the contributions of observation and theory to the development of our science. Great credit is given to theoretical workers, especially in modern times, but the pioneer and indeed essential observational work is stressed. The address constitutes a summary of astronomical development, and on account of the condensation required to cover so wide a field in so short a space, it is necessary for the reader to go slowly to completely grasp all the points raised. At the end a strong plea is made for the development of astronomical observation along lines specially selected, with instruments specially designed for the solution of definite problems, rather than the blind carrying on of observation along traditional lines with any equipment which may be to hand. Professor Plaskett's experience both at Victoria and Harvard has impressed on him the importance of observations, although he also played a part in the development of theory to fit the results of observation, and we congratulate him on securing a grant of £2,400 for new equipment for the observatory under his control. The lecture can be well recommended to those who have some knowledge of modern development in astronomy, but much of it would be rather unintelligible to the outsider.

OBITUARY.

R. T. A. Innes: 1861-1933.

Robert Thorburn Ayton Innes died suddenly in London on 13th March, 1933. A foundation member of this Society, he was its second President, succeeding the late Mr. S. S. Hough. At the time of his death he was Director of the Society's Computing Section.

Dr. Innes' interest in astronomy dated back to his school days. Before he reached the age of eighteen he had been elected a Fellow of the Royal Astronomical Society. In these early years his interest in astronomy was chiefly mathematical, and he published some papers dealing with the secular perturbations of the earth.

In the early 'nineties he emigrated to Sydney, N.S.W., where he was engaged in the wool trade. There he met several Australian astronomers, among them Mr. W. F. Gale, whose friendship was the turning point in Innes' career. Mr. Gale lent him an old 6¼-inch refractor made by Cooke. With this instrument, very roughly mounted equatorially, Innes began his great life work in astronomy—his investigation of double stars in the Southern heavens.

His loaned telescope was without circles, so in his early efforts he confined his search to the vicinity of naked-eye stars. Thirty hours' search by these crude means revealed no less than twenty-six new doubles! Innes published these results on 10th December, 1894, and within a few months a second list was ready. These later discoveries were made with a silver-on-glass reflector, ground and mounted by a New South Wales amateur.

His experiences with two small telescopes decided Innes on a complete change of vocation. He wanted bigger instruments—the biggest available, and these were only to be found in an established and well-equipped observatory. Innes wrote to Sir David Gill, offering his services. All Sir David could give him was temporary clerical work at a very small salary. His Australian income was more than adequate, his prospects healthy. But the call of astronomy was too strong. He accepted Gill's offer.

At the Royal Observatory of the Cape, Innes' researches were confined to his leisure hours, for officially he was Secretary, Librarian and Accountant. Even so, his work was prodigious. By 1898—two years after his arrival—he had discovered 280 new double stars. The following year he published the first "Reference Catalogue of Southern Double Stars," which deals with 2,140. This was followed by other works on double and variable stars, and also by the "Revision of the C.P.D.," a monster task that led to the discovery of many variables, red stars, and stars of large proper motions.

Early in 1903, Innes left the Cape for Johannesburg, to establish a Transvaal observatory for meteorological and time services. He had, however, no intentions of abandoning his astronomical investigations. The observatory had no telescope, so on occasions he would borrow a 4-inch refractor from a local amateur. Later, largely through the good offices of Sir David Gill, at the Cape, and Dr. T. Reunert, in Johannesburg, he obtained some useful equipment. With this he commenced his classic series of observations of the eclipses, transits and occultations of Jupiter's satellites.

Further instruments followed, and in 1910, when the Transvaal was merged in the Union of South Africa, the last Transvaal Government had already placed an order with the firm of Sir Howard Grubb for a 26-inch refracting telescope. By a sardonic twist of fate, Innes was to be debarred from crowning his life work with a protracted series of observations with this great instrument. Vexatious delays and the Great War intervened. The great telescope was for his successor.

Dr. Innes will be remembered as far more than a savant of science. He was, in the truest sense, a man of the world. He was full of enthusiasm and unexpected inhibitions. Even in his astronomy these peeped out. Few astronomers would have dared to claim for the blink-microscope what Innes claimed for it. Yet with it he detected Proxima Centauri, the nearest known star, and for this spectacular discovery he is best known to the man in the street. On the other hand, he was sometimes conservative, where most other astronomers had little difficulty in accepting the interpretation of evidence. For example, he once publicly expressed his doubts about deductions based on interferometer observations.

Then, too, he would sometimes indulge in the most daring hypotheses. His attempt to explain ice ages by the visitations of comets will be fresh in the minds of many.

Besides astronomy, he had many interests. The writer has particular reason to remember his interest in chess. At chess he played a game of almost ruthless precision. Could he have devoted his whole time to it he might have become a world master. As it was, he competed in South African Championship

Tournaments with distinction. After he abandoned chess he took up basic English, and from the interest he aroused sprang basic Afrikaans!

Besides all this, Dr. Innes took an almost ingenuous interest in politics, and dabbled in invention. Perhaps he did more than dabble, for a form of bioscope projection which he invented may lead to material improvements, particularly in small halls. It was in connection with this invention that he was in London at the time of his unexpected death.

He leaves a host of friends. Among them are the many famous oversea astronomers, labouring in our midst, whom Innes attracted to this country by his personality, and by his advocacy of the surpassing excellence of South African star-shine.

Science will remember Innes as a great astronomer—the greatest of all authorities on Southern variable stars. South Africa will remember a great citizen, joyous in outlook, selfless in service, and well beloved.

ASTRONOMICAL SOCIETY OF SOUTH AFRICA.

Session 1932 - 1933.

Annual Report of the Council.

In presenting its Annual Report, the Council is able to record a year of steady progress in the work of the Society. The roll now includes 131 members, 5 associates, 6 honorary members and 2 members emeriti. The increase in the number of members is due largely to the addition of a new Centre to the Society — the Natal Centre — as a result of the action of the late Natal Astronomical Association at its annual meeting on the 2nd September, 1932, in passing the following resolution:

> " That this Association agrees to amalgamate with the Astronomical Society of South Africa, and that the Rules and Bye-laws drawn up by the sub-committee and submitted to this meeting be adopted."

This, together with the resolution at our last Annual Meeting,

> " That in the event of the adoption at the Annual Meeting of the Natal Astronomical Association of the proposal for union with this Society, the said Natal Astronomical Association be incorporated in this Society

in terms of Article VIII. as amended, and shall be established as the Natal Centre of the Society,"

brings to fruition the negotiations referred to in our last two Annual Reports and widens the boundaries of the Society.

The Council has met four times during the year. those members who are eligible under Article VII. (iii.) of the constitution being represented by alternates. As a result of the representations of the deputation, referred to in the last Annual Report, which met the Improvements and Parks Committee, your Council is pleased to be able to inform you that the City Council agreed to take formal transfer of the Herschel Reserve and to register the right of way to it, and has informed the Society that authority has been issued for the necessary action to be taken. Certain improvements, including the transfer of the plate bearing the English inscription from the South side to the East side and the insertion of a glass plate in its place in order to render visible the granite cylinder erected by Sir John Herschel, have been executed.

In view of the approach of the centenary of the arrival of Sir John Herschel at the Cape to carry out his survey of the Southern skies, your Council has considered the possibility of initiating centenary celebrations. The Royal Society of South Africa has been approached, and it is hoped that a joint committee may be established to deal with the matter. It has been ascertained that other societies and institutions would give their support to the scheme.

The Society has suffered a heavy loss by the departure of its President, Dr. Spencer Jones, whom it congratulates on the signal distinction accorded him by his appointment to the post of Astronomer Royal. Dr. Spencer Jones has done much work in the interests of the Society as President, Editor, and Member of Council, and our deep appreciation of these services was given expression on the 8th February, when a meeting was held to bid him farewell before his departure for Greenwich. The Council has elected Dr. Spencer Jones an honorary member.

The Council is pleased to be able to record that Dr. Jackson, his successor as H.M. Astronomer at the Cape of Good Hope, has given the Society his support by becoming a member.

During the year Vol. III. No. 2 of the Journal has been published, and also a pamphlet containing an account of the farewell presentation to Dr. Spencer Jones, on which occasion the Society was honoured also by the presence of its distinguished member, General Smuts, who took the chair and made the presentation.

The work of the observing sections has been steadily continued throughout the year, and Mr. Forbes is particularly to be congratulated on sharing with Mr. Dodwell, of Australia, the honour of the discovery of Comet 1932*n*. This makes Mr. Forbes' fourth discovery. In addition to the activities of the established observing sections, some of our members have observed occulations and have forwarded the times to Dr. Comrie. Others have been making observations of the Zodiacal Light and the Gegenschein and forwarding monthly reports to "Popular Astronomy." The Council has extended the scope of the Society by creating a Mathematical Section, whose activities shall include the organisation of the computational work of members of the Society.

It is with deep regret that the Council has to record the death of Dr. Innes, Dr. Bennetts, and Messrs. Rayleigh and Gohl. Dr. Innes was a foundation member of the Society and has taken an active part in its development, having served the Society as President and as Director of the Computing Section.

REPORTS OF SECTIONS.

For the Year ended 30th June, 1933.

COMET SECTION.

A good deal of useful work has again been done this year by members of the Section in searching for new and for returning comets. We wish to acknowledge our indebtedness to the Royal Observatory in favouring us with copies of cables received intimating new discoveries and we have to acknowledge the value of the Union Photograph Star Maps which enable us to state the position of objects with much accuracy.

Periodic Comets.

The B.A.A. Handbook for 1932 contained predicted ephemerides of nine comets. Of these, all were observed except Schorr, Neujmin (2) and Temple (1). The 1933 book contains the ephemerides of four comets:—Pons-Winnecke, Finlay, Giacobini-Zinner and Wolf (1). The first and third have already been observed. About Giacobini-Zinner, Dr. Crommelin says, "Its orbit makes a near approach to that of the Earth and meteors may be looked for next October. . . . It passes the descending node on October 9th." Members of the Society should look out for meteors about that date and make a careful note of them.

New Comets.

Comet 1932*f* (Newman), mag. 13. This comet was discovered at Lowell Observatory on June 20th by Newman. Its position then was R.A. $15^h\ 37^m$, North Dec. 7° 56′. It never became a bright object.

Comet 1932*g* (Geddes). The discovery of this comet was mentioned in our last report. It was well observed by our members when it was in the southern skies and observers in the north can still see it. On account of the long time it has remained under observation it has become a well known object and much interest is now attached to it on account of the opinion held by some authorities that its orbit is hyperbolic. The fact of it remaining under observation for so long a period is a very valuable factor in deciding this important point.

Comet 1932 (Peltier-Whipple). A new comet was found at Delphos, Ohio, by Mr. L. Peltier, and at Harvard (independently by photography) by Dr. F. L. Whipple. It reached naked eye visibility and had a tail 2 degrees long. Its period was found to be 302 years.

Comet 1932*n* (Dodwell-Forbes), mag. 10. A new comet was discovered by Mr. G. F. Dodwell, Government Astronomer at Adelaide, Australia, on December 17th. It was also discovered independently by your Director at Hermanus, South Africa, on December 15th, when he noticed it in the field with Fomalhaut. Its position at discovery was R.A. $22^h\ 54^m$, South Dec. 30° 15′, The following orbit is by Dr. Crommelin:

T	1932 Dec. 30·5285 U.T.	
w	327° 23′ 4″·4	
☊	78° 12′ 22″·4	1932·0
i	24° 20′ 30″·5	

Its period has not yet been definitely established but appears to be about 250 years. At perihelion its magnitude increased to nearly 8, and it was well observed by our members. *Popular Astronomy* published photographs of it taken by Prof. van Biesbroeck on December 20th and on January 20th. In the first it is described as round, about 3′ in diameter, condensed in the centre without stellar nucleus. In the second it showed a large, round coma and a long straight tail 1° in length which had a narrow axis exactly away from the Sun, with lateral streamers making a narrow fan.

Comet 1933*a* (Peltier), mag. 8, was discovered by Mr. L. Peltier at Delphos, Ohio, on February 16th. Its position at discovery was R.A. $22^h\ 48^m$, North Dec. 62°. It travelled rapidly south but though several of our members tried to see it no report of its having been seen has reached us.

The Donohoe Comet Medal for the Pacific was awarded to Messrs. H. E. Houghton and G. E. Ensor for the discovery of Comet 1932*b*, and to the writer for the independent discovery of Comet 1932*n*.

A. F. I. Forbes, *Director.*

VARIABLE STAR SECTION.

Your Director is pleased to be able to report a successful year, 3,233 observations having been recorded. These are divided between the two members of the Section as follows:

H. E. Houghton, F.R.A.S.	2,014	observations of	86	variables.
G. E. Ensor	1,219	,, ,,	126	,,

Your Directors' observations were unfortunately interrupted through illness, in December, January, and February. Mr. Houghton's splendid contribution, however, went far towards bringing the year's total into line with those of previous years.

It is to be regretted that the work of the Section is still being carried on by two members only. An earnest appeal is made for new members to join the Section. Charts and all information will gladly be supplied on application to the Director of the Section.

Your Director's thanks are due to the Union Astronomer, Mr. H. E. Wood, and to Dr. W. H. van den Bos, the Chief Assistant at the Union Observatory, for much help and encouragement, also for the notes with reference to the spectrum of Nova Pictoris, and the micrometer measurements of the nova and its companions.

Notes.

Nova Pictoris.—Apart from a few small fluctuations this nova has not decreased in brightness to any appreciable extent during the past apparition. Its present magnitude is 8·7. The following notes with reference to the spectrum of the nova, extracted from a recent letter from Dr. Spencer Jones, have been kindly supplied by Mr. H. E. Wood:

"It does not appear as though the spectrum has altered much since the previous year. The ionised helium line at 4,686 looks to have become more intense, relatively to the other lines. It is interesting to see that the two strong unknown lines at 6,087 and 5,722 show no signs of fading yet. Theoretical investigations have so far been quite unsuccessful in providing a clue as to their origin. Nova Pictoris seems almost to have reached a stationary magnitude, and it looks as though it is not going to fade to the pre-burst magnitude."

The following notes and measurements of the nova and its companions have been supplied by Dr. van den Bos, by kind permission of the Union Astronomer:—

Observer — Dr. J. Voute, 23½-inch, Lembang.

AB, 1932·85 79°·0 0″·78, one night 'very difficult; perfect definition.' Voute has no measure of AC later than 1930·164.

Observer — Van den Bos, 26½-inch, Johannesburg.

AB	1933·18	69°·1	1″·27	4 nights, Mag. 9·1 — 13·4
AC	1933·18	234°·2	1″.43	4 nights Mag. 9·1 — 14·1

Notes. — In 1932 the image was still markedly nebulous, though the extent of the visible nebula was small (probably due to loss of brightness), more South than North of the central nucleus A. The companions B and C were well outside this nebula, but very faint and hard to measure.

In 1933 the nebulous character of A had become almost unnoticeable; B was very faint but still measurable. C, however, was almost a glimpse-object, and its measures are hardly better than guess work.

Observer — Finsen, 26½-inch, Johannesburg.

AB	1933·15	73°·6	1″·20	2 nights, Mag. 9·0 — 12·2
AC	1933·15	230°·2	1″·26	2 nights, Mag· 9·0 — 13·5

Notes. — Difficult; B and C well outside the greatly diminished nebulosity of A. (I may add that Finsen estimates very faint companions systematically brighter than I do.)

(Signed) W. H. VAN DEN BOS."

S. Apodis. — This irregular variable was below 13·4 mag. in July, 1932. On August 19th it had increased in brightness to mag. 13·0, and in June, 1933, to 10·3, which is nearly the normal maximum.

RY Saggitarii. — This irregular variable remained fairly steady at maximum from July, 1932, till October, 1932, after which date proximity to the Sun prevented further observations. When picked up again by Mr. Houghton in March, 1933, the star had decreased in brightness to mag. 9·0, and in April, 1933, to mag. 11·4.

RY Saggittarii is now increasing in brightness again, and at the end of June, 1933, was mag. 9·2.

G. E. Ensor, *Director.*

CAPE CENTRE.

Nineteenth Annual Report, 1932 - 1933.

Your Committee in presenting this, the Nineteenth Annual Report of the Centre, have to record its continued activity.

Membership.

Eight additions have been made to the roll of membership during the year. There have been twenty-three deletions, owing to death, resignation, transfer and other causes. Members who are resident in Natal and are now attached to the recently established Natal Centre, have ceased to be members of this Centre. The membership is now as follows:

Ordinary members, 85; Members Emeriti, 2; Associates, 5 — a total of 92 against 104 at the close of last year.

Meetings.

During the period under review there have been nine Ordinary meetings, all of which have been well attended. The addresses and papers presented at these meetings have maintained the standard of interest to which members have become accustomed, and included the following:

"Setting up a Reflecting Telescope and Grinding and Figuring the Speculum": Mr. G. O. Neser, M.Sc.

"Stars of the Evening Sky in July": Mr. H. W. Schonegevel.

"The Movements of the Earth": Mr. R. Watson.

"Objects of Interest in the Evening Sky" (October): Mr. A. W. Long, F.R.A.S.

"A Voyage through Space" (Popular Lecture): Rev. Andrew Graham.

"The Amateur's Telescope": Inst. Capt. M. A. Ainslie, R.N., F.R.A.S.

"History of the Royal Observatory at Greenwich": Sir Frank Dyson, F.R.S.

"Meteoritic Craters": Mr. J. W. Nankivell, B.A., F.R.G.S.

"Retrograde Motion of Planets": Capt. D. Cameron-Swan, F.R.A.S.

"Variation of the Date of the Equinox": Mr. R. Watson.

"On Constructing a Hut for Reflecting Telescopes": Inst. Capt. M. A. Ainslie, R.N., F.R.A.S.

By the invitation of Mr. Bertram F. Jeary the Anuual Observation Meeting was held at his observatory at Muizenberg in February. Weather conditions proving unfavourable for observing, the meeting resolved itself into one of discussion of various topics, the chief feature being an exposition by Dr. J. K. E. Halm, of the possible causes of Ice Ages.

Discovery.

The Committee have pleasure in recording the discovery of a comet by a member of the Centre. Comet 1932*n* was discovered by Mr. A. F. I. Forbes at Hermanus on December 15th, 1932. It was discovered independently by Mr. G. F. Dodwell, Government Astronomer at Adelaide, Australia, on December 17th.

Finance.

The financial statement will show that while the Centre has been able to pay its way, its finances are not as satisfactory as heretofore. The Centre is thus sharing in the general depression. Members are urged to remit their subscriptions promptly, so as to facilitate the conduct of the affairs of the Centre.

Articles in the Press.

Notes with charts of the sky continue to be published in the "Cape Times" monthly, these being contributed by Mr. A. W. Long.

Articles in Afrikaans are published in "Die Burger"—contributed by Mr. T. Mackenzie.

The publication of the foregoing is greatly appreciated by members and the public generally.

Financial Statement for the Year ending 30th June, 1933.

RECEIPTS.	£ s. d.	PAYMENTS.	£ s. d.
Balance in hand at 30th June, 1932	2 14 8	Contributions under Article IX. of Constitution	23 4 8
Subscriptions:		Rent of Meeting Room	10 0 0
Arrears .. £2 2 0		Rent of P.O. Box ..	1 5 0
Current Year 42 10 6		Subscription to Astronomical Society of the Pacific	1 10 6
In Advance 1 16 9	46 9 3	Typewriting and Stationery	4 4 6
Commission on Cheques	0 4 6	"Cape Times," and Postage to Country Members	5 1 9
Debit Balance	1 10 5	Advertising	0 15 0
		Secretary's Expenses ..	2 8 6
		Treasurer's Expenses .	0 15 0
		Lecturer's Expenses ..	0 10 0
		Bank Charges	1 3 11
	£50 18 10		£50 18 10

JOHANNESBURG CENTRE.

Annual Report, 1932-33.

The year ending 30th June, 1933, in the affairs of the Johannesburg Centre of the Society, seems to have been following the sun-spot cycle, as it has been somewhat uneventful.

The loss of Dr. R. T. A. Innes, whose lively interest in, and wide knowledge of, Astronomy rendered his attendance at our meetings so stimulating and informative, is keenly felt; and the passing of his genial personality, ever ready with assistance or advice on matters of interest to members, leaves a gap that will not readily, if ever, be filled.

Visits were paid to the Union and Yale Observatories by good numbers of the Centre, and our thanks are extended to the Union Astronomer and Dr. Alden for their hospitality on these interesting and instructive occasions.

Difficulty was experienced in getting any formal papers for the other meetings, so none were given; but a question night and discussions on matters of astronomical interest were greatly appreciated at the meetings held, when our members took a lively part in dealing with the matters brought forward.

Statement of Income and Expenditure for the Year ended 30th June, 1933.

INCOME.	£ s. d.	EXPENDITURE.	£ s. d.
To Balance on hand 1st July, 1932	22 4 11	By Transfer to Headquarters	7 10 0
„ Members' Subscriptions	10 18 6	„ Rent, S.A.R. & H. Salaried Staff Association (15/- and 7/6)	1 2 6
		„ Donation to Dr. Spencer Jones' Presentation	2 2 6
		„ S.A. Ribbon & Carbon Co., for Stationery	1 1 0
		„ Postages and Petty Cash (includes £1 1s. for Membership to British Astronomical Association)	1 12 6
		„ Bank Charges	0 10 6
		„ Balance Carried Forward	19 4 5
	£33 3 5		£33 3 5

ASTRONOMICAL SOCIETY OF SOUTH AFRICA.

Statement of Income and Expenditure for the Year ended 30th June, 1933.

INCOME.	£ s. d.	EXPENDITURE.	£ s. d.
To Balance, 30th June, 1932	22 17 4	By Printing Journal, Vol. 3, No. 2	27 9 0
„ 50% Subscriptions (Cape Centre) ..	23 4 8	„ Printing Journal Supplement	5 0 0
„ Sale of Journals ..	1 6 3	„ Printing and Stationery	2 18 0
„ Sale of Sundial ..	1 13 5	„ Postages	2 8 10
„ 50% Subscriptions (Johannesburg Centre, 1931-32) ..	7 10 0	„ Sundries	1 10 0
		„ Rent	0 15 0
		„ Cheque Book and Commission	0 7 9
		„ Balance Carried Forward	16 3 1
	£56 11 8		£56 11 8

Examined and found correct.
E. J. STEER, Hon. Auditor.
20th July, 1933.

W. H. SMITH,
Hon. Treasurer.
30th June, 1933.

ASTRONOMICAL SOCIETY OF SOUTH AFRICA.

Officers and Council, 1933-34.

President: D. G. McIntyre, F.R.A.S., Civil Service Club, Cape Town.

Vice-Presidents: J. Jackson, M.A., D.Sc.; A. Rossiter, M.A., Ph.D.; C. L. O'B. Dutton.

Hon. Secretary: H. Horrocks, M.A., F.R.A.S., Royal Observatory, Cape of Good Hope.

Hon. Treasurer: W. H. Smith, "Arum Villa," Plumstead.

Members of Council: H. E. Houghton, F.R.A.S.; A. W. Long, F.R.A.S.; A. Forrest; H. E. Wood, M.Sc,, F.R.A.S.; D. L. Forbes, F.R.A.S.; C. F. Wickes, F.R.A.S.

Alternate Members of Council: R. R. Pratt, B.Sc., A.M.I.C.E.; J. W. Nankivell, M.A., F.R.G.S.; W. Andrews; C. E. Peers; Capt. Cameron-Swan, F.R.A.S., F.R.P.S., F.S.A. (Scot.); H. W. Schonegevel.

Directors of Observing Sections.

Comet: A. F. I. Forbes, M.I.A., "Blairythan," Main Road, Hermanus, C.P.

Mars: B. F. Jeary, "Villa Carina," Alexander Rd., Muizenberg.

Variable Stars: G. E. Ensor, Pretoria Hospital, P.O. Box 201, Pretoria.

Mathematical: D. C. Alletson, B.A., Diocesan College, Rondebosch.

Hon. Editor: J. Jackson, M.A., D.Sc., F.R.A.S., Royal Observatory, Cape of Good Hope.

Hon. Librarian: D. C. Burrell, Pinelands, Cape Town.

Committee of Cape Centre.

Chairman: R. Watson.

Vice-Chairman: H. W. Schonegevel.

Hon. Secretary: A. Menzies, P.O. Box 2061, Cape Town.

Hon. Treasurer: R. R. Pratt, B.Sc., A.M.I.C.E., "Rochester," Boundary Road, Rondebosch, C.P.

Hon. Librarian: D. C. Burrell.

Hon. Auditor: E. J. Steer.

Committee: H. Horrocks; D. G. McIntyre; A. W. Long; B. F. Jearey; H. E. Houghton.

Committee of Johannesburg Centre.

Chairman: H. E. Wood, M.Sc., F.R.A.S.

Hon. Secretary: J. D. Stevens.

Hon. Treasurer: A. Forrest.

Committee: Miss Troughton; Dr. Alden; and Messrs. Beamish, Worssell and Holmes (of Johannesburg) and Messrs. Burroughs and Ensor (of Pretoria).

Committee of Durban Centre.

Chairman: J. Bennet Rumford.

Vice-Chairmen: H. J. S. Bell and H. Roadknight.

Committee: Mrs. J. Grix; Messrs. F. T. Fox and J. Willis.

New Members, 1932-33.

Mrs. I. D. McDougall, 50, Rissik Street, Krugersdorp.

Instructor Capt. M. A. Ainslie, R.N., B.A., F.R.A.S., F.R.M.S., "Greentrees," Sunnybrae Road, Rondebosch.

Mr. H. C. Davis, Orange Street, Ceres.

Mr. H. E. Krumm, Leeuwendal Crescent, Cape Town.

Dr. J. Jackson, Royal Observatory, Cape.

Mr. D. C. Alletson, Diocesan College, Rondebosch.

Mr. J. W. Nankivell, Diocesan College, Rondebosch.

Dr. J. A. J. van Rensburg, Diocesan College, Rondebosch.

The Society acknowledges the receipt of publications, etc., from the following:—

University Observatory, Babelsberg, Berlin; Harvard College Observatory; Lick Observatory; Talence Observatory, France; University Observatory, Kasan; Union Observatory, Johannesburg; British Astronomical Association, Glasgow Branch of the British Astronomical Association, Sydney Branch of the British Astronomical Association; New Zealand Astronomical Society; Philosophical Society, University of Durham; South-West Africa Scientific Society; Argentine Astronomical Society; Argentine Association of Friends of Astronomy; Antwerp Astronomical Society; Dr. J. Comrie.