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Alexandrian Astronomy

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This in fact turned out to be a re-discovery of some aspects of early Greek astronomy as practised by astronomers who worked in the Great Library of Alexandria from about 300 BC to about AD 150. Aristarchus used a lunar eclipse to estimate the size of the Moon and later Hipparchus improved on this. The same geometry was again used much later by Copernicus in "De Revolutionibus".

If the size of the Earth is calculated using Eratosthenes method and an eclipse of the Moon is observed/photographed, then the distance to, and the size of, the Moon can be found using some simple equipment, some straight forward geometry, some mathematics, basic trigonometry and a little ingenuity.

[The full text of this paper appears in the *Centrepiece* of this issue.]

Living Inside the Cosmic Egg

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Imagine waking up one morning and finding yourself trapped inside a hollow easter egg! You cannot see out - and you cannot get out. It seems a weird analogy, but, on a cosmological scale, it is true.

Our view of the entire observational universe is a sphere. We are at the centre and the radius is about 15 billion light years. As we look outwards through the sphere, we look back in time. But the sphere is contained within an opaque shell, because the very early universe was opaque, not transparent- as it has been ever since. The shell exists in time, not in place. It is a horizon, not a physical structure. Nevertheless we can never see through it, or go through it. We appear trapped inside this cosmic egg!

* *This is the first instalment in a series of abstracts of papers delivered at the national Symposium. Full versions of available papers appear separately, elsewhere in this issue.*

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CENTREPIECE

June 1999

Workshop module for high school teachers

Alexandrian Astronomy Today

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1. Introduction

The advent of a lunar eclipse in April 1996 seemed an opportune time to develop an exercise to calculate the size of, and distance to, the Moon as part of the South African Astronomical Observatory's Science Education Initiative.

This in fact turned out to be a re-discovery of some aspects of early Greek astronomy as practised by astronomers who worked in the Great Library of Alexandria from about 300 BC to about AD 150. Aristarchus used a lunar eclipse to estimate the size of the Moon and later Hipparchus improved on this. The same geometry was again used much later by Copernicus in "De Revolutionibus".

If the size of the Earth is calculated using Eratosthenes method and an eclipse of the Moon is observed/photographed, then the distance to, and the size of, the Moon can be found using some simple equipment, some straight forward geometry, some mathematics, basic trigonometry and a little ingenuity.

2. Geometry of lunar eclipses

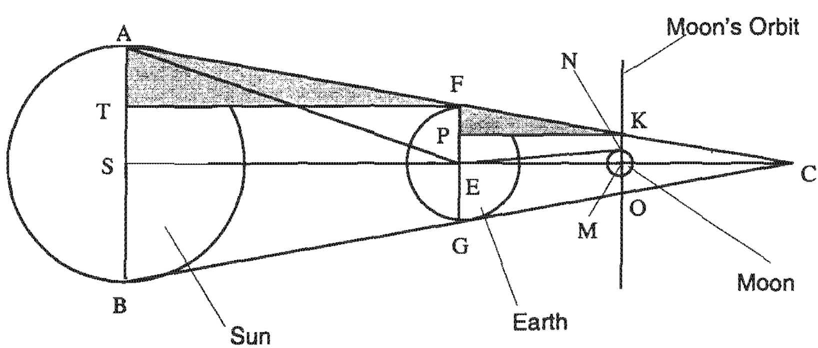

Figure 1. Geometry of a lunar eclipse, showing the Earth–Sun distance (ES=D), Earth–Moon distance (EM=d), Sun's radius (AS=R), Earth's radius (EF=r), Moon's radius (MN=a) and the radius of the Earth's shadow (MK=s).

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In figure 1, the triangle FGC represents the Earth's shadow and line KMO the Moon's orbit. Assume that the Sun is n times further from the Earth than the Moon, i.e.

$$\mathrm{SE} = n\,\mathrm{EM}, \quad \text{or} \quad \mathrm{D} = n\,d \tag{1}$$

As can be seen during an eclipse of the Sun, the Sun and Moon appear to be the same size, i.e. subtend more or less the same angle in the sky, and so ∠AES = ∠MEN; therefore, triangles ASE and MNE are similar. They don't look similar in figure 1, because the diagram is not drawn to scale. Then, using (1) above:

$$\mathrm{AS} = n\,\mathrm{NM}, \qquad \mathrm{R} = n\,a, \qquad n = \frac{\mathrm{AS}}{\mathrm{MN}} = \frac{\mathrm{R}}{a} \tag{2}$$

Triangles ATF and FPK are also similar and so:

$$\frac{\mathrm{TF}}{\mathrm{PK}} = \frac{\mathrm{AT}}{\mathrm{FP}}, \qquad \frac{\mathrm{SE}}{\mathrm{EM}} = \frac{\mathrm{AS}-\mathrm{TS}}{\mathrm{FE}-\mathrm{PE}}, \qquad \frac{\mathrm{SE}}{\mathrm{EM}} = \frac{\mathrm{AS}-\mathrm{FE}}{\mathrm{FE}-\mathrm{MK}} \tag{3}$$

Using the assigned symbols and substituting (1) and (2) into (3) we get:

$$\frac{D}{d} = \frac{n\,d}{d} = \frac{R-r}{r-s}, \qquad n = \frac{n\,a-r}{r-s} \tag{4}$$

which if rearranged gives:

$$a\left(1+\frac{s}{a}\right) = r\left(1+\frac{1}{n}\right) \tag{5}$$

So, to find the distance to the Moon, d, values for r, n, a in terms of d and the ratio

$$\frac{s}{a} \tag{6}$$

need to be found. This ratio can be found during an eclipse of the Moon. The radius of the Moon a can be found in terms of d by measuring the angular diameter of the Moon. Using Eratosthenes' method, the Earth's radius r can be found and n, well, that can be overcome!

3. Measurements from the lunar eclipse

The Moon is photographed during an eclipse to get an image that looks similar to Figure 2. This occurs when the Moon is entering the umbra. It might be best to take a series of photographs with a long telephoto lens or telescope so that there is a selection to choose from. The photograph is enlarged to that accurate measurements can be made from it as shown below. If there isn't a convenient eclipse to photograph, it is also possible to copy a picture from the press or from a previous eclipse.

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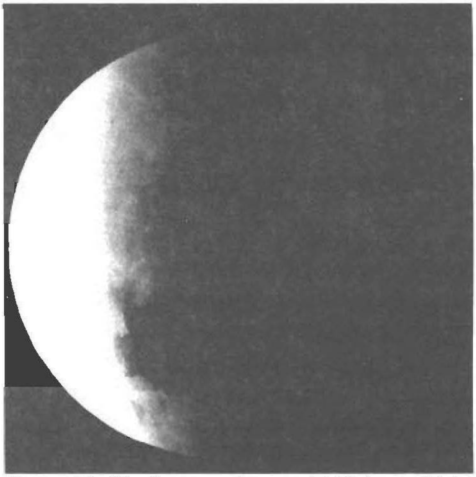

Figure 2: The lunar eclipse of 1996 April 3/4, photographed by Case Rijsdijk (SAAO) with a Celestron C8.

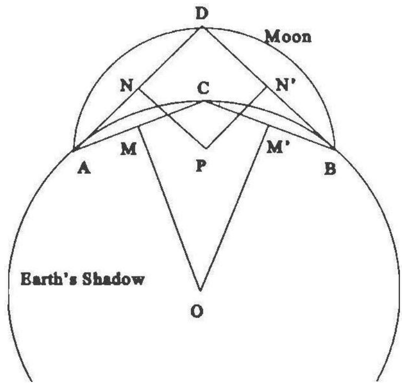

Figure 3: Geometrical determination of the ratio of the Moon's radius to the radius of the Earth's shadow.

The photograph is needed to find the ratio of the radius of the Moon to that of the Earth's shadow. There are several ways this can be done; probably the best is using the geometry of the circle and a scale drawing. By making as large a copy of the photograph as possible (and photocopying it), the points C and D (see figure 3) are marked off so that they are as far apart as possible along an imaginary radius extended.

A and B are the points of intersection of the circles. The perpendicular bisector of the chord of a circle will pass through the centre of that circle. N is the midpoint AD and N′ is the midpoint of DB. The perpendiculars from these points meet at P. DP would then be the radius of the Moon. Similarly, OC is the radius of the Earth's shadow at the distance of the Moon's orbit and then the ratio

$$\frac{OC}{DP} = \frac{s}{a} \qquad (7)$$

4. Angular size of the Moon

This can be done either directly or indirectly.

Method 1: Directly

A slider is made to move smoothly along a metre rule. A small ball bearing (about 6mm in diameter) is mounted on the slider. A disk with a small hole in it is fixe dat one end as shown in figure 4. The rule is rested on something firm and the Moon is sighted through

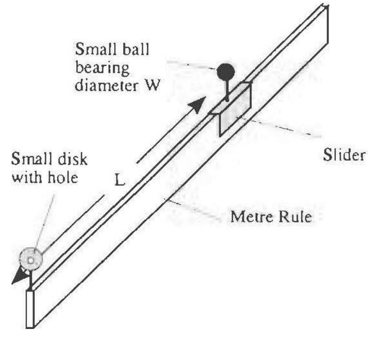

Figure 4

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the small hole in the fixed disk. The slider is now moved so that the ball bearing *exactly* covers the Moon. The angle subtended by the ball bearing is now the same as that subtended by the Moon:

$$\tan\alpha = \frac{W}{L} \qquad \therefore\ \alpha = \tan^{-1}\frac{W}{L} \tag{8, 9}$$

Great care needs to be taken, as this result is crucial to obtaining an accurate result.

Method 2: Indirectly

Since the Sun and the Moon appear to be the same size (subtend the same angle), as shown during a solar eclipse, it is possible to construct a good size pinhole camera and get a reasonable image of the Sun which can be marked. The ratio of the *image size* to *image distance* will again give the required angle α.

5. The size of the Earth

Eratosthenes' experiment can easily be repeated using some shadow sticks. Another school/college/university is found on a north-south line and at least 500km away. Once contact has been established, the length of the shadow of a perpendicular stick (figure 5) is measured at each place at the same time. Make sure that the stick is perpendicular by using a simple plumb line made up from a "bulldog" clip, some string, and a lead sinker. For example, two places W and T have been chosen (figure 6) and at each place the length *L* of the shadow and the height *H* of the stick is measured at say 12h00. The angle ϕ can then be found from:

$$\tan\phi = \frac{L}{H} \tag{10}$$

Similarly, the angle β at T can be calculated. It is then simple geometry to show that

$$\theta = \beta - \phi \tag{11}$$

The size of the Earth

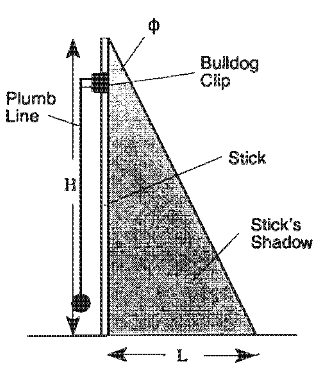

Figure 5

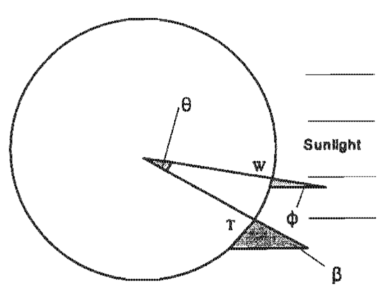

Figure 6

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The distance WT = S is found from an atlas and then by simple proportions:

$$\frac{S}{C} = \frac{\theta}{360} = \frac{S}{2\pi r} \tag{12}$$

where C is the Earth's circumference and r its radius.

Some examples of places that could be used in Southern Africa are:

Lusaka (Zambia) – Bulawayo (Zimbabwe)
Cape Town (SA) – Windhoek (Namibia)
Port Elizabeth (SA) – Bloemfontein (SA)
East London (SA) – Johannesburg/Pretoria (SA)
Durban (SA) – Harare (Zimbabwe)
Grahamstown (SA) – Welkom (SA)
Gaberone (Botswana) – Bloemfontein (SA)

The two main sources of error in this experiment are:

• due to diffraction, there is some difficulty in seeing a clear shadow and also in determining exactly where the shadow ends; it usually covers about 2cm or so.
• the stick not being perpendicular.

With some care, the second can be overcome. The first is best sorted out by making the stick as long as possible and securing a 'T' piece at the top. Accuracy will also be improved if as many different groups as possible do this part of the experiment and average out the result.

6. Working out the Earth–Moon distance

Use equation (5) and substitute the following values that have been found:

1: from equation (8) the angle subtended by the Moon, α, was obtained. This was the angle subtended by the diameter; for the radius half this value is required, i.e.

$$\tan\frac{\alpha}{2} = \frac{a}{d} \qquad \therefore \quad a = d\tan\frac{\alpha}{2} \tag{13}$$

2: the ratio s / a which was obtained from the eclipse and using equation (7)

3: the value obtained for r using the shadow sticks and equation (12).

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This leaves n (see equation 5). Now Aristarchus realized that the Sun was further away from the Earth than the Moon was, but he did not know how much further. He tried to work it out using geometry but the value he got, 20, was too small.
However, if it is assumed that n is very large, then the ratio $1/n$ is very small, and can be neglected.

There are now two ways to calculate a and d:

Using equations (13) and (5):

$$d \tan\left(\frac{\alpha}{2}\right)\left(1+\frac{s}{a}\right) = r$$

The value for d can now be found, since all the other values are known. Once d has been found, a can be calculated using:

$$a = d \tan \frac{\alpha}{2}$$

The second method of calculating a and d gives a minimum value for the radius of the Moon:

$$a\left(1+\frac{s}{a}\right) = r \qquad \therefore \quad a = \frac{r}{1+\frac{s}{a}}$$

The value for a can now be found since all the other values are known. Once a has been found, d can be calculated using equation (13). This is an alternative way of doing it and probably better!

7. Alexandrian values

Hipparchus used the following values:

$$\frac{s}{a} = \frac{8}{3}$$

$$\alpha = 31'$$

This gives for $\alpha/2 = 15.5'$, and $a = d/220$ (approximately). He used Eratosthenes' value for r (6500km) and calculated the distance to the Moon $d = 390\ 000$km. The diameter of the Moon he calculated as 3 516km.

This doesn't compare too badly with presently accepted figures of 384 404km for the mean distance to the Moon (ranging between 358 020km and 405 936km), and 3 476km for the diameter of the Moon.

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8. Modern day values

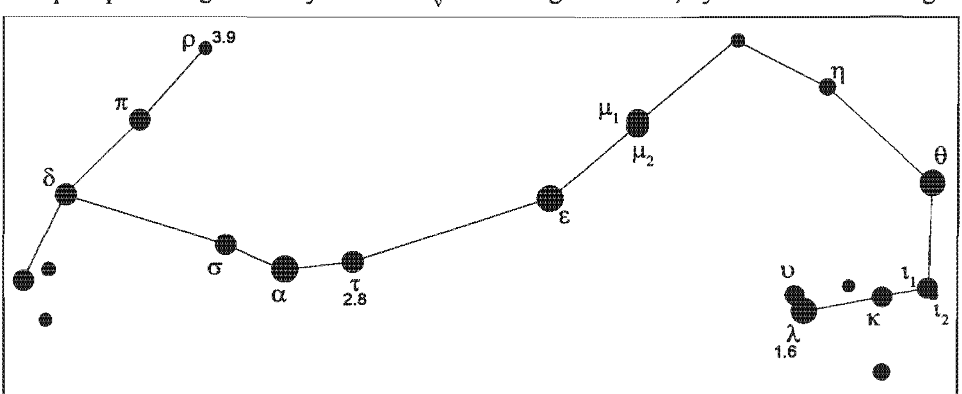
Several local schools took part in the project last year, and the following two results are fairly typical:

	School 1	**School 2**
Data collected		
Ratio of Earth's shadow/Moon (s/a)	2.67	2.67
Angular diameter of the Moon	26′	31′
Earth's radius	6 383km	6 384km
Calculated values		
Distance to the Moon from Earth	457 731km	386 161km
Diameter of the Moon	3 461km	3 476km

These results, whilst not accurate, do give reasonable figures that are not too far from the true values. It is fairly obvious that the critical result in this experiment is the angular size of the Moon and the statistical mean from a large number of readings would appear to be the best way to obtain an accurate result.

NAKED EYE ASTRONOMY

With winter well on its way, deepsky observers are gearing up to explore the fabulous regions of Scorpius and Sagittarius. The easily recognizable Scorpion constellation also affords a good opportunity to test one's skills at estimating stellar magnitudes. The diagram below shows the familiar outline of the constellation, with the Bayer designations for selected stars. Their visual magnitudes lie between 1.6 and 3.9, represented by lambda (λ) and rho (ρ) Scorpii, with tau (τ) Scorpii representing a midway value of m_v=2.8. Using these stars, try and estimate the bright-



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