The Period-Luminosity Law – how an amateur was first to determine the distance to delta Cephei

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Discussion

The prototype of the Cepheid variables, delta Cephei, was discovered by the deaf and dumb observer John Goodricke in 1874. He lived to the ripe old age of 22 years. It was he who first explained the variable brightness of Algol as being due to two stars revolving around each other, so that they alternately eclipsed and transited each other – certainly a very revolutionary idea in 1784!

Goodricke was also the first with the explanation that delta Cephei (the fourth brightest star in the constellation Cepheus) owed its variability in brightness to pulsations of the star, i.e. it alternately expanded and contracted.

It was Henrietta Swan Leavitt (1868-1921) who had distinguished herself in the determination of photographic magnitudes of stars at Harvard College Observatory, who first got the idea that there may be a relationship between the brightness of a Cepheid variable and the length of its period of variability. So in 1905 she went to Chile to study the Cepheids in the two Magellanic Clouds. In the Harvard Observatory Circular No.173 of 1912 she published her findings in an article entitled "Periods of 25 variable stars in the Small Magellanic Cloud".

The graphs in Figures 1 and 2 are reproductions from tracings of Henrietta Leavitt's original graphs. In the first graph (Figure 1) she plotted apparent visual magnitudes against periods of variability from maximum to minimum brightness; the magnitudes ranged from 10.5 to 17 and the periods from 1 day to 125 days.

In the second graph (Figure 2) she plotted apparent visual magnitudes against the logs of the periods. This revealed the linear relationship between apparent visual magnitude and the logarithms of the periods. It also showed that the apparent visual magnitude was proportional to the logarithms of the periods of variability. This meant that greater brightness was concomitant with the larger logs of the periods.

Miss Leavitt reasoned that since the Magellanic Clouds are very far away, their stars being of magnitude 10-12 and fainter, it would be correct to suppose that the stars in the Small Magellanic Cloud, for example, are all equally far from the Earth because the stars on the near side of the Cloud are approximately equally as far as the stars on the far side of the Cloud. It can be likened to the crowd at a rugby match at Twickenham in London – the spectators on the south side are equally as far from us in Johannesburg as those on the further northern side. There would therefore be a linear relationship between apparent magnitudes and absolute magnitudes – a constant difference. The apparent magnitudes would therefore be indicative of the intrinsic brightness of the stars in the Small Magellanic Cloud. This would apply, only because the stars in the Small Magellanic Cloud were a compact group, very far away.

Henrietta Leavitt was thus able to formulate the law: **the absolute magnitude (or luminosity) of a Cepheid variable is proportional to the logarithm of its period of variability**. In order to apply this law to all Cepheid variables, wherever they may be found, it became necessary to standardize the apparent magnitudes with the absolute magnitudes of the Cepheids in the Small Magellanic Cloud – there would be a constant difference between absolute magnitude and apparent magnitude.

Much work was done by H Shapley, M L Humason and E P Hubble to determine the absolute magnitudes of Cepheids having spectra similar to those in the Small Magellanic Cloud. Shapley also made use of radial velocities and proper motions in order to gain some idea of the distances involved.

After some years the difference between apparent magnitude (m) and absolute magnitude (M) was determined. This is the distance modulus of the stars, and from it the distance of the Cepheids could be determined from the formula:

$$m - M = -5 + 5\log D \qquad (1)$$

where D = distance in parsecs.

It would have been easy if there were some Cepheids nearer than 300 light years, which is the limit to which the trigonometrical method of distance determination can go, but there were none.

When the absolute magnitude (M) was known, the distance of the Cepheid could be calculated from the formula:

$$M = m + 5 - 5\log D \qquad (2)$$

Using the absolute magnitudes determined by Shapley, Humason and Hubble, it appeared that the distance of the Large Magellanic Cloud was 120,000 light years and that of the Small Magellanic Cloud 130,000 light years, thus proving Miss Leavitt's supposition about the distances of the two Clouds.

Cepheids all over the Milky Way showed that the diameter of the Milky Way was at least 50,000 light years – the Magellanic Clouds thus had to be far outside the Milky Way. The 100 inch Hooker telescope on Mount Wilson showed Cepheids in the Andromeda Nebula (as it was then known) to be at least 850,000 light years distant. The galaxies were thus found to constitute the building blocks of the universe.

W H W Baade, using the new 5 metre telescope of Mount Palomar, came to the conclusion that there was something wrong with the Cepheid scale and that it was incorrect to lump all Cepheids, whatever their periods, into one law. He showed that the very short period RR Lyrae variables are all A-type giants with absolute magnitudes 0.8, the classical Cepheids of periods 2 to 30 days are F and G giants, and the long period Mira types are red giants of types K and M. There should therefore be separate laws for (a) RR Lyrae variables, (b) classical Cepheids with periods 2 to 30 days, and (c) red giants of long periods.

Using these new scales it became apparent that the universe was at least twice as large as had been supposed and that the Large Magellanic Cloud is 166,000 light years distant, the Small Magellanic Cloud 205,000 light years and the Andromeda Galaxy, as it now became to be known, 2,200,000 light years away, this value corresponding to that which had been indicated by the supernova of 1885 which had been discovered in the heart of the Andromeda Galaxy.

While writing my first book "Ontsluier die Heelal" I obtained a list of absolute magnitudes and periods of Cepheids drawn up by A S Sandage and G A Tammann of Mount Palomar, as published in the Astrophysical Journal 1969, 157.683. I used 12 stars with periods ranging from 1.95 to 13.62 days, all being classical Cepheids and discarded those of longer periods. The data is shown in Table 1.

The first two columns in the Table are the Sandage and Tammann raw data. The third column contains the logs of the periods. The graph in Figure 3 is a plot of the absolute magnitudes against the logs of the periods. Figure 3 reveals a linear relationship between absolute magnitude and log of the period. We want to derive the equation to the straight line which best fits the data. We can determine this equation by making use of the method of least squares. The equation to the straight line has the form:

$$y = a + bx \qquad (3)$$

where a is the intercept on the y-axis and b is the slope of the line. We thus have to find the values of a and b. This is done as follows:

1. add the absolute magnitudes in the first column, giving sigma y (Σy)

2. Find the mean of y by dividing Σy by 12 (-42.6/12 = -3.55)
3. Add the logs in column 3 and divide by 12 to find the mean (= 0.76616)
4. Subtract the mean of the logs (using 3 significant figures = 0.766) from each of the logs. This gives the x column
5. Find the sum of the x values (= -0.002). This is sigma x (Σx)
6. Multiply the values in the y-column (absolute magnitude) by the values in the x-column, giving values xy. The sum of this column $\Sigma xy = -1.651$
7. Square the values in the x-column, giving x^2, and add, giving $\Sigma x^2 = 0.608$

Since the mean of the logs has been taken as 0.766 instead of 0.76616, the amounts subtracted on each line from ℓ are 0.00016 too small. Therefore corrections have to be made as follows: Σx has to be increased with 12 x 0.00016 (i.e. 0.02), thus giving $\Sigma x = 0$ as it should be. Σxy has to be corrected by -42.6 x -0.00016 (i.e. +0.007). The correct value of Σxy is thus -1.651 + 0.007 = -1.644. The correction in x^2 does not affect the third decimal place.

In the method of least squares the sum of the deviations of the points from the line of best fit will be 0 AND the sum of the squares of the deviations must be a minimum. In this case we have:

$$a = \frac{\Sigma y}{n} = \frac{-42.6}{12} = -3.55 \qquad (4)$$

and

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{-1.644}{0.608} = -2.70 \qquad (5)$$

Therefore the equation to the straight line of best fit is:

$$\begin{aligned} y &= a + bx \\ y = M &= -3.55 - 2.70x \\ \text{or} \quad &= -3.55 - 2.70 \log(P - 0.766) \\ \bullet\ M &= -3.55 - 2.70(-0.766) - 2.70 \log P \\ M &= -1.48 - 2.70 \log P \qquad (6) \end{aligned}$$

We can draw this straight line by finding the coordinates of points A and B in Figure 4.

For point A: substitute x = 0.3, $\bullet M = -1.48 - 2.70(0.3) = -2.29$
For point B: substitute x = 1.2, $\bullet M = -1.48 - 2.70(1.2) = -4.72$

In order to find the 95% confidence limits above and below the straight line, within which 95% of cases will fall, we have to find the sum of the deviations of the points above and below the straight line. This sum Σd will be zero. Then we have to find the squares of the deviations. Their sum Σd^2 comes to 0.3081. The square root of this, 0.160, is the standard deviation. Twice the standard deviation, 0.32, is the 95% confidence limit. In our case it comprises 0.32 magnitude in Figure 4.

Any amateur astronomer can use the equation derived here to calculate the absolute magnitude of any Cepheid with period between 2 and 13 days. So let us amateurs take delta Cephei as our guinea pig. The period of delta Cephei is 5.366 days. Substitution into (6) gives:

$$\begin{aligned} M &= -1.48 - 2.70 \log(5.366) \\ &= -1.48 - 2.70(0.729) \\ \bullet\ M &= -3.45 \end{aligned}$$

The absolute magnitude of delta Cephei is thus -3.45. We know its apparent visual magnitude to have a mean of 4.18, so we are able to calculate its distance from (2)

$$\begin{aligned} M &= m + 5 - 5\log D \\ \bullet\quad \log D &= \frac{m - M + 5}{5} \\ &= \frac{4.18 + 3.45 + 5}{5} \\ &= 2.53 \end{aligned}$$

The antilog of 2.53 is 338.8. This is the distance of delta Cephei is parsecs. In light years its distance is 338.8 x 3.26 = 1104 light years.

I derived this value in 1989. In the catalogues delta Cephei's distance was given as 630 light years. I stuck to my guns and published my book showing the distance to be 1100 light years. In 1993 G Gatewood and his team at Allegheny Observatory, using their Multichannel Astrometric Photometer, measured the parallax of delta Cephei with unparalleled accuracy and obtained a value of 0.003 arc seconds. This gives a distance of delta Cephei of 1/0.003 = 333 parsecs. This comes to 333 x 3.26 = 1085 light years. So, they did very well!